Problem 5 : Cryptic Message

With the algorithm, we can find x such that:

$$rx + qy = \gcd(r, q) = 1.$$

So

$$rx \equiv 1 \mod q.$$

We see

$$\sum_i m_i b_i = \left(\sum_i m_i w_i r\right) + q \cdot L_1$$

We can multiply this by x:

$$\left(\sum_i m_i w_i r x\right) + q \cdot L_2 = \left(\sum_i m_i w_i\right) + q \cdot L_3$$

Now since $q > \sum_i w_i$ and the $m_i \in \{0, 1\}$, we can do modulo q:

$$\left(\sum_i m_i w_i\right) + q \cdot L_3 \equiv \sum_i m_i w_i$$

Now since we know the w_i, this is the Lunchbox problem from earlier.